

Stress Concentrations Optimisation Process for Engineering Structures

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ABSTRACT

Prediction of fatigue life is a critical issue in the design of aerostructures. Stress concentrations are crucial in this connection. The paper proposes a design optimisation methodology aimed at achieving a longer and safer working record for a structure over its lifecycle, based on the redistribution of material to balance the effect of stress concentrations. The idea of design optimisation in situations where primary failure is driven by stress concentrations is worthwhile and has the potential to offer design engineers a better view in the design structures subjected to fatigue loading and an opportunity for better utilisation of material with its associated economic benefits.

Keywords: Stress concentrations; Structural optimisation design; Design methodology

1. INTRODUCTION

The engineering and manufacturing industries are operating in rapidly changing markets, and facing severe competitive pressures, and increasing customer demands for higher quality products at lower costs. Shorter product lifecycles also place increased pressure on the product introduction process. This means that it is those manufacturers that have the ability to engineer their products right first time, through the use of the most appropriate materials, structures and technology will be the market leaders. A culture of design analysis rather than prototype and experimental test is highly expected. The availability of design optimisation tools especially structural and topological optimisations is the key requirement among these expectations.

Strong and lightweight structures have always been a major area of interests for both academia and industry. The work reported in this paper has logically developed out stress concentration analysis research and the development of application software for the design of aerostructures and highly stressed machine systems.

The paper proposes a design methodology for optimisation with the objective of minimising the effect of stress concentrations. It aims to achieve a longer and safer working record for a structure over its lifecycle, based on redistribution of material to balance the effect of stress concentrations. Cantilever, Single-Beam-Bridge and L-shaped Structure, which often occur in many design situations, are presented in the paper as single-feature (no holes, notches, shoulder, countersinks, etc.) case studies to demonstrate the algorithms and

design methodology. Multiple-feature case studies will be published in the future research.

2. STATE OF THE ART

There are many methods available for structural and topological optimisation. Michell was an early pioneer creating the optimality criteria (OC) [1], which has been continuously developed up to present. It is applied to design minimum weight pin-joined frameworks subject to stress constraints. OC has been an influential advance in the development of optimisation technology; however, because of the amount of data processing required it could only be applied to the simplest of problems [1]. Many years later the advent of the computer allowed it to be exploited to a much greater extent.

Evolutionary structural optimisation (ESO) was first introduced by Xie and Steven in 1992 [2-5]. It has become a foundational and important practical methodology for structural and topological optimisation. Bidirectional evolutionary structural optimisation (BESO) [6] introduced the notion of flexibility in terms of removal and/or addition of material. Additive evolutionary structural optimisation (AESO) [7] performed the optimisation procedure from the starting point of a minimal structure to carry a load spectrum regardless of stress level. The addition of material is approximately positioned in the highly stressed areas to reduce the effect. However, there is no dynamic link between the original structure and the produced optimum surface shape.

Metamorphic Development (MD) is a multi-objective methodology devised in the Engineering Design Centre at Cambridge University around 1999 to deal with minimizing structural compliance and weight subject to stress and deflection constraints [8, 9]. MD enables the structural growth and degeneration to occur simultaneously during the iteration cycle. Elements that are less effective in contributing to the strength and stiffness of the structure are removed and new elements that are more effective are added to the structure. However, unlike AESO, MD has no argument for defining the maximum domain before beginning the optimisation process.

Computer aided shape-optimisation (CAO) [10, 11] is based on the idea that biological structures grow naturally into a homogeneous surface stress state. It has successfully generated optimum topologies that adapt to the biological loading by changing their surface profile: however, this method is very much restricted to the simulation of a thin layer of the biological structure due to the narrow options which have been chosen in finite element (FE) construction.

The method of homogenization [12-17] introduces the concept of continuum topology optimisation as redistribution of perforated composite materials with variations of boundary orientation and density. This method results in an optimal structure constructed in an anisotropic state with various sizes of holes and various densities. A limitation of this method is that the results produced can sometimes be impractical, and counter to more general geometry and structural requirements.

The density function approach for optimal topology design uses linear programming [18]. It treats element density as a continuous variable to formulate the topology design problem by sequential linear programming. The advantage of this approach is its capability to handle various problems with multi-objective functions and multiple design criteria. However, it involves a large number of variables to ensure the sufficiency for sensitivity analysis and optimisation, and therefore requires careful analysis and expertise.

The Harmony Search (HS) meta-heuristic algorithm [19] can be effective for structural optimisation. It uses a stochastic random search based on the harmony memory considering rate (HMCR) and pitching adjusting rate (PAR). The HS algorithm comes out of the musical

process of stochastic searching for a perfect state of harmony and guides a global search rather than a gradient search. However, extra vectors that are generated after consideration of HMCR and PAR could possibly relax the rigidity and stiffness of the structure. The ability to solve building size optimisation problems is very impressive. The mass of the optimized structure tends to give better solutions compared with those obtained from conventional approaches.

While, the above methods have found many useful engineering applications, they are not ideally suited to the problems of structural design where stress concentration and fatigue problems are the main concerns. Prediction of fatigue life is a critical issue in the design of aerostructures.

The main problem arises out of the complexity of the structural configuration, involving many potential fatigue sites. Design features such as holes, slots, lugs etc can produce very high local stresses. These are crucially important in aerostructural design - stress concentration problems cause fatigue failure.

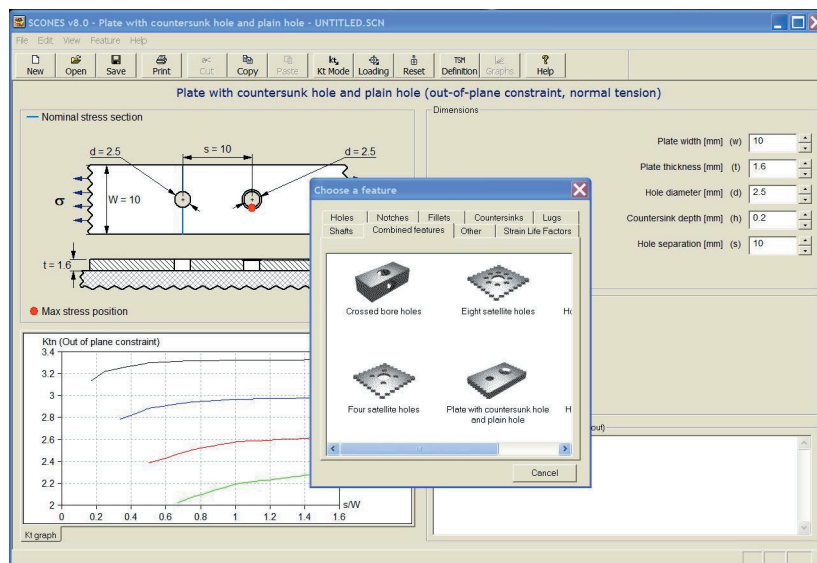


Figure 1 An example of the SCONES interface to show features interaction for a countersink hole and a plain hole.

Determination of the so-called stress concentration factor (Kt) is essential for the correct design of components subjected to fatigue-inducing loading. Based on research in stress concentration and fatigue analysis in aerostructural design BAE SYSTEMS in collaboration with the University of Hull has developed a Stress Concentration Expert System (SCONES) [20, 21]. The SCONES software provides rapid and precise solutions for aerostructural design and fatigue engineers. If the local stresses are too high and fatigue lifetime is compromised, alternative design geometries/materials are considered. However, SCONES does not support structural optimisation, merely the calculation of the stress concentration factor (Kt). An example of the SCONES interface is shown in Figure 1. The paper now goes on to consider how optimisation technologies maybe used to support such design work.

3. THEORETICAL CONSIDERATIONS

Structural optimisation is an organized process for improving parameters following a certain design concept but without changing the concept itself. The material is utilized within the allowed design space subjected to a certain objective (e.g., maximum or minimum stress, minimum mass, maximum stiffness, highest frequency). The means of structural optimisation by which a structural design evolves from an initial to a final form can be described as an optimisation process wherein attempts are made to minimize costs (say) or maximize benefits while satisfying design criteria reflecting performance and fabrication requirements [22, 23].

The approach taken in this paper to structural optimisation is founded on finite element approximations for elastic deformation of static engineering problems. The structural optimisation generally requires the definition of applicable load spectrum and restrains. Specialized FE software, e.g. Ansys, is normally used to assist the implementation of the objectives in the design environment.

Consider a thin element lying in a global coordinate of x, y plane, loaded by in-plane forces or moments applied in the x, y plane at the boundary. For this reason the stress components σ_z, τ_{xz} and σ_y are assumed to be equal to zero. Therefore the stress components σ_x, σ_y and τ_{xy} are functions of x and y only. The differential equations of equilibrium together with the compatibility equation for the stresses (x, y and xy in a plane of elastic structural materials) is presented as [24]:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \bar{p}_{vx} = 0 \quad (1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \bar{p}_{vy} = 0 \quad (2)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\sigma_x + \sigma_y) = -f(\nu) \left(\frac{\partial \bar{p}_{vx}}{\partial x} + \frac{\partial \bar{p}_{vy}}{\partial y} \right) \quad (3)$$

where $\bar{p}_{vx}, \bar{p}_{vy}$ denote the components of the applied body force per unit volume in x and y directions and $f(\nu)$ is a function of Possion's ratio. For plane stress $f(\nu)$ is equal to $1 + \nu$. For a constant body force,

$$\frac{\partial \bar{p}_{vx}}{\partial x} = \frac{\partial \bar{p}_{vy}}{\partial y} = 0$$

hence the equation (3) becomes:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\sigma_x + \sigma_y) = 0 \quad (4)$$

To determine the stress distribution in two-dimensional problems with constant body forces the equations shown above should be sufficient. However even very simple loading

conditions may result in a biaxial stress system, therefore the von Mises criterion as a theory of stress analysis, produces a reasonably good interpretation, is adopted in this research.

4. OPTIMISATION METHODOLOGY

In developing the methodology it has been assumed that lower order rectangular elements are sufficient for this research. There are no general guidelines for choosing the optimum element for a given engineering optimisation problem, but this type of element generates acceptable accuracy with low computing time. It reduces difficulties in interpretation and in model construction. Lower order linear and quadratic elements contain fewer interior nodes; therefore the complexity of the meshing can be reduced to a certain degree, whilst facilitating the addition of more elements and more degrees of freedom to the initial domain. The rectangular elements with sidelines parallel to the global coordinates are easily developed in the Stress Concentration Optimisation Process for Engineering Structures (SCOPES) to interpret the function of the whole domain [25]. Where the global coordinates $x_0 = 0$, $y_0 = 0$ and the local coordinates are x_i and y_j , finite node number n assigned to the element e is m, j is the number of nodes in the local coordinates.

The discretization of the initial domain would decide the element size where the gradient of the von Mises stress distribution field is expected to be maximal with fine tuned convergence. After this discretization, all the elements and nodes will be identified and stored in a parameter table with a distinguished number in a compressed sequence. The nodal solution will be recalled as an identity to classify a location from which the local coordinates will be registered for further analysis. Identification of the maximum nodal stress continues until the element is located on the boundary of the sub domain. This process does not stop until the sub domain meets the required criteria for the design. Once the Addition is finished, the volume of the initial domain becomes the reference and used for the next step, which is the Reduction stage of SCOPES. With traditional design, engineers are not necessarily aware of how much excess material is being employed. The structure resulting from the following optimisation process is likely to be superior or at least verifying previous designs.

The nodal solutions for all the elements are stored in a parameter table to be recalled to identify the minimum von Mises stress on the boundary. In the Reduction stage, once again the nodal coordinates are used to identify the location for the removal of the elements until the criteria for the reduction is met. The criteria for Addition and Reduction are kept as simple as possible in the application of the methodology. A schematic flow chart for SCOPES is shown in Figure 2.

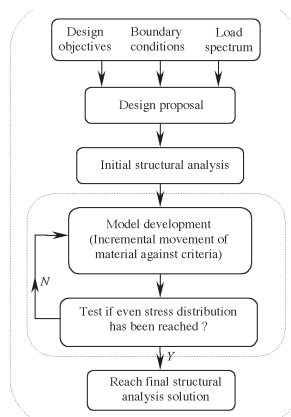


Figure 2 Flow chart for stress concentration optimisation process

To facilitate the process, an assistant procedure is called to produce the discrete number ξ_e of the element with maximum or minimum von Mises stress in order to construct the Addition or Reduction. The discrete number ξ_e is in a range of $1 \leq \xi_e \leq 4$, which is determined by the number of the elements surrounding the node that holds the maximum or minimum von Mises stress. Figure 3 depicts the configurations for the possible geometrical boundaries in SCOPES.

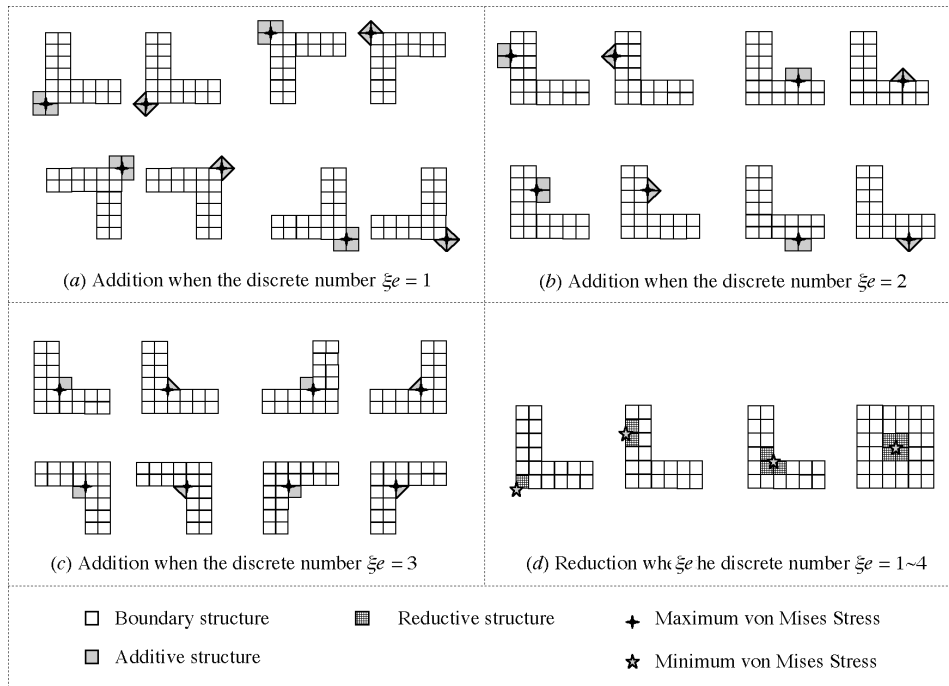


Figure 3 Stress concentration optimisation process, (a)-(c) Addition criteria and (d) Reduction criteria.

It is worth noting that triangular elements might well produce similar results. (As mentioned above rectangular element is used in SCOPES as the unit for Addition and Reduction) If triangular elements were used at the boundary area (to compensate for the potential limitations of rectangular elements) the structural optimisation could possibly be more effective. Therefore, future research could be concerned with application of triangular elements.

5. EXAMPLES

5.1. CANTILEVER

The first example illustrates a classic optimisation problem - a horizontal cantilever beam connected to a constrained wall and subjected to a constant force vertically applied at its end. The parameters are as follows: The material used in this case is assumed to be Al alloy with a Young's modulus of 70 GPa and a Poisson's ratio of $\nu = 0.27$. A force of 1 N is vertically applied to the top surface of the beam, which has a width of 1 mm and a length of 11 mm . The initial geometry is equally divided into 704 rectangular elements.

The criteria are set up without change to the loading point and the constraints of the cantilever. A limit of 95% is placed on the reduction of the maximum von Mises stress from the initial structural analysis. The elements carrying the minimum von Mises stress are the candidates to be removed from the structure. The Addition and Reduction processes were set to work within the boundaries of 115% to 25% of the initial volume. In addition, all the elements of the cantilever in contact with the wall will be constrained.

Figure 4 shows the volume change to the Cantilever resulting from the application of SCOPES. A consequence of the process strategy is that model development begins with the creation of voids within the structure. As can be seen in the figure as the optimisation process progresses the number of voids moves from 7 at 115% to single void at 25% of the initial structural volume. The effect of the optimisation process on the level of maximum stress associated with each of the structural design iterations can be seen in Table 1. The Structural Design Efficiency shown in the Table is defined as the maximum stress in the generated design divided by the maximum stress associated within the initial structure, expressed as a percentage.

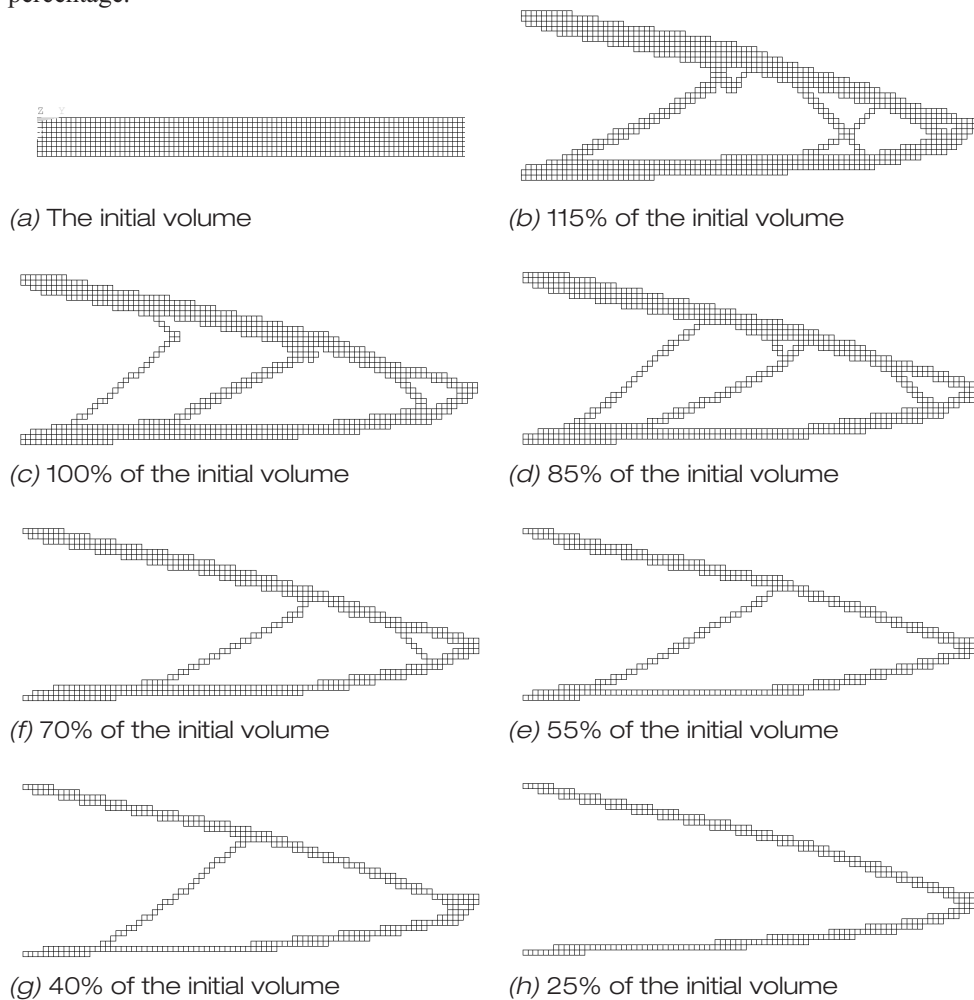


Figure 4 Optimisation for Cantilever, (a)-(h) volume changes from 115%-25%.

Table 1 'Structural Design Efficiency' for a Cantilever Design

Volumetric Change (%)	115	110	85	70	55	40	25
Structural Design Efficiency (%)	15.1	17	17.7	27.9	29.8	38.4	51.8

5.2. SINGLE-BEAM-BRIDGE

The second example is similar to the frequently occurring Michell-type structural problem [26] to be found in many civil engineering applications. To keep consistency with the first example, the parameters - dimensions and the material properties - of the beam are unchanged. A load of 1 N will be moved to a central position of the beam and the ends will be constrained completely. The structure has exactly the same number of rectangular elements.

As before, the criteria are set up without change to the loading point and the constraints of the Single-Beam-Bridge. A limit of 95% is placed on the reduction of the maximum von Mises stress from the initial structural analysis. The elements carrying the minimum von Mises stress are the candidates to be removed from the structure. Again, the Addition and Reduction processes were set to work within the boundaries of 115% to 25% of the initial volume. Additionally the elements at both ends of the beam will be constrained.

Figure 5 shows the volume change for the Single-Beam-Bridge from the application of SCOPES. As can be seen in the figure as the optimisation process progresses the number of voids moves from 5 at 115% to a single void at 25% of the initial structural volume. At around 55% of volume change the optimisation process creates an extra void in the lower part of the structure and consequentially the upper part begins to disappear with decreasing volume. The effect of the optimisation process on the level of maximum stress associated with each of the structural design iterations can be seen in Table 2.

Table 2 Structural Design Efficiency for Single-Beam-Bridge Design

Volumetric Change (%)	115	110	85	70	55	40	25
Structural Design Efficiency (%)	16.6	27.9	34.3	40.9	64.4	84.3	133.5

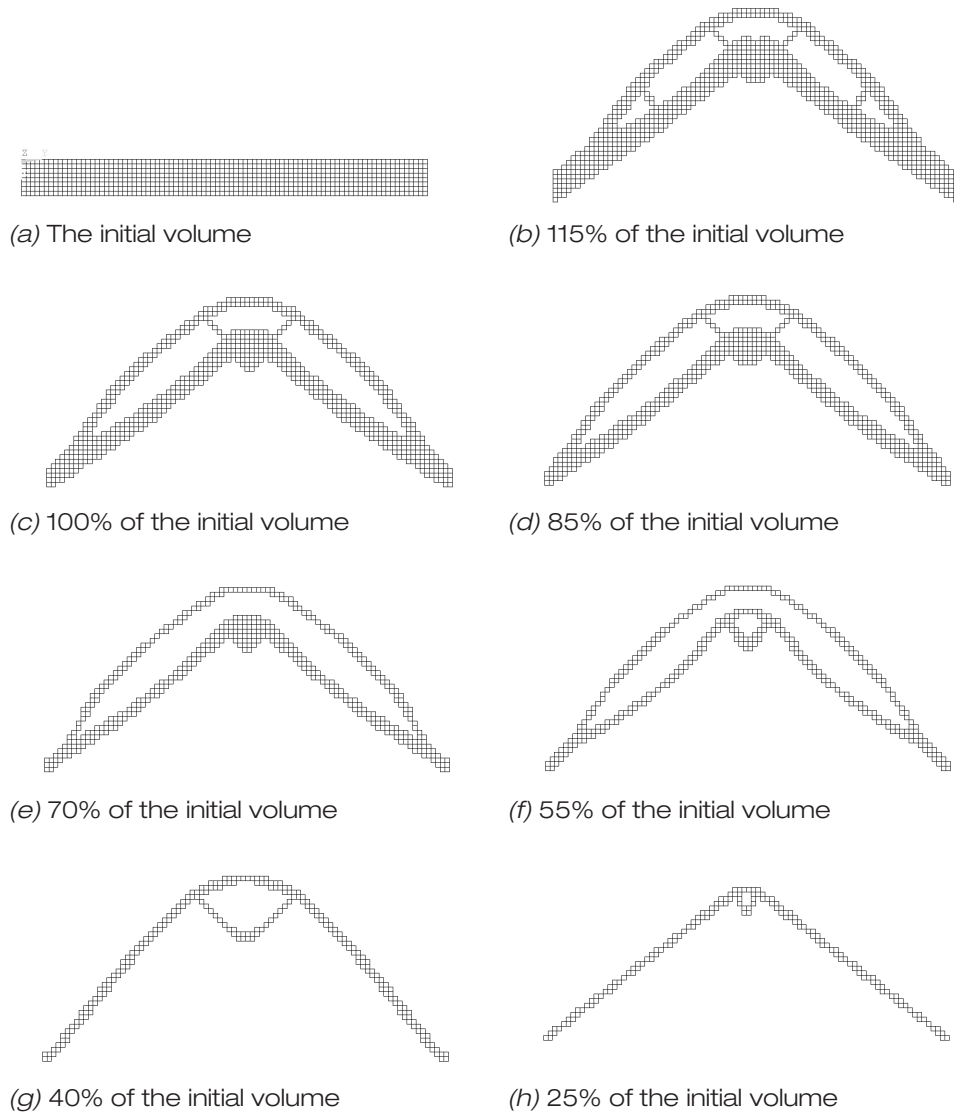


Figure 5 Optimisation for Single-Beam-Bridge with fixed ends and central load, (a)-(h) volume changes from 115%-25%.

5.3. L-SHAPED STRUCTURE

The third example explores the combination of multiple forces or bending moments and represents a more complex problem. The need to identify the nodal stresses distribution and the use of a substantial number of elements put the pressure on the capabilities of *ANSYS 9.0*.

The example is essentially two beams with identical dimensions (width of 1 mm and a length of 11 mm) joined in a right angle*. The material properties are kept as same as those used in the example 1. (A force of 1 N is horizontally applied on the top right of the vertical beam and another force of 1 N is vertically applied on the top right of the horizontal beam).

* The beams are joined to produce an equally sided section and symmetrical along the 45° axis and free with no constraints.

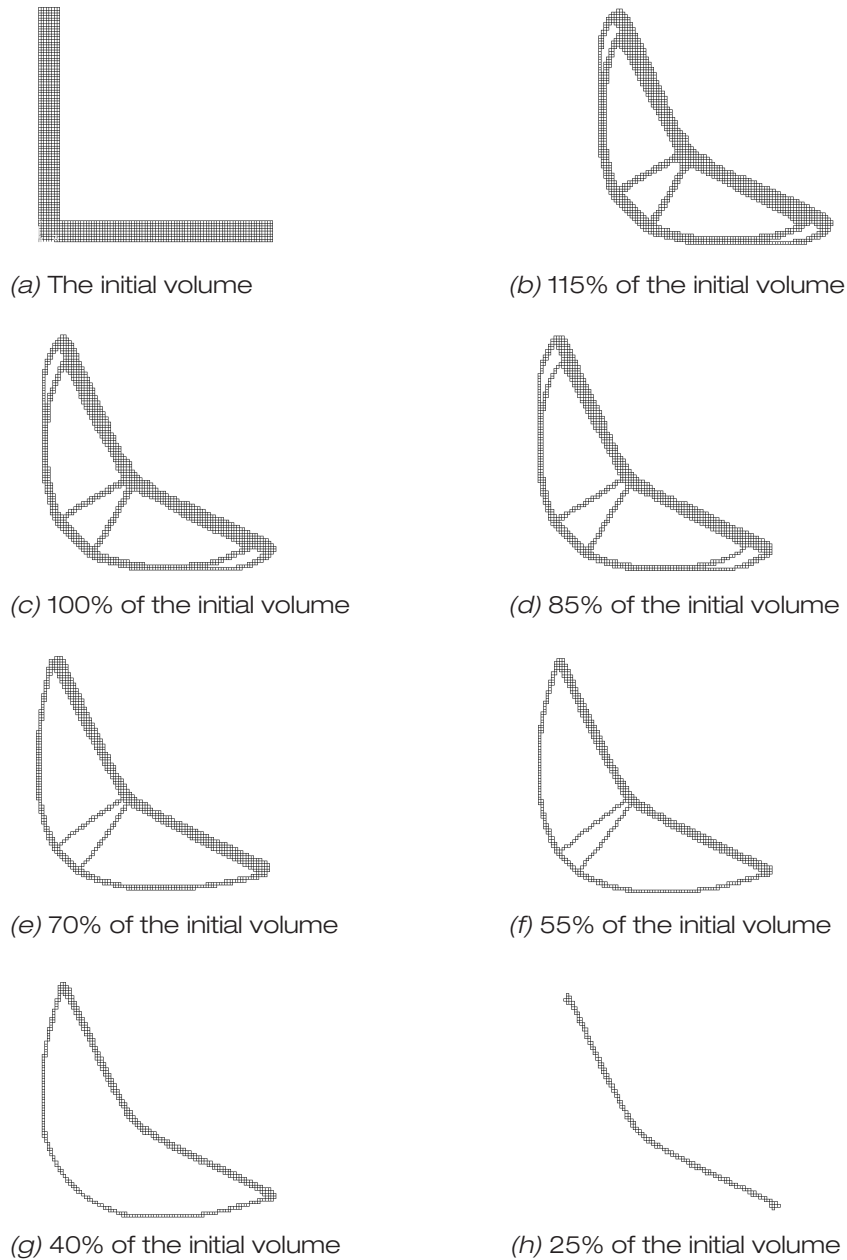


Figure 6 Optimisation for L-shaped Structure, (a)-(h) volume changes from 115%-25%

The criteria are set up without change to the loading point of the L-shaped Structure with no constraints at all. A limit of 95% is placed on the reduction of the maximum von Mises stress from the initial structural analysis. The elements carrying the minimum von Mises

stress are the candidates to be removed from the structure. The Addition and Reduction processes were set to work within the boundaries of 115% to 25% of the initial volume. The structure has been purposely divided into divisions of 1344 rectangular elements.

Figure 6 shows the volume change for the L-shaped Structure resulting from the application of SCOPES. As can be seen in the figure as the optimisation process progresses the number of voids moves from 5 at 115% to no void at 25% of the initial structural volume. The effect of the optimisation process on the level of maximum stress associated with each of the structural design iterations can be seen in Table 3. This example suggests that SCOPES has the capability to deal with complex loading modes applied to irregular geometry.

Table 3 Structural Design Efficiency for L-shaped Structure Design

Volumetric Change (%)	115	110	85	70	55	40	25
Structural Design Efficiency (%)	6.7	7.5	8.7	14.4	15.3	37.8	87.1

6. DISCUSSION AND CONCLUSIONS

The SCOPES methodology for structural design optimisation in engineering applications has been presented. The results suggest that it has the capability to create improved structural designs in situations where high local stresses may occur.

The results have shown that lower order rectangular elements have been sufficient, although further research will consider the application of alternative element types in the future. For example, the inclusion of triangular elements could provide smoother models of surface geometry for the SCOPES process. The variety of problem types and volume change has shown that the SCOPES process can be flexible in dealing with a wide range of stress concentration problems.

The idea of design optimisation in situations where primary failure is driven by stress concentrations remains worthwhile and has the potential to offer design engineers a better view in the design of structures subject to fatigue loading and provides the opportunity for better utilisation of material with its associated economic benefits. However, care must be taken when using such approaches to ensure that all design requirements are satisfied. There is not much material left in the structure at 25% of initial volume.

In the above design optimisation methodology no account has been taken of manufacturability, and an optimised design coming out of the process may not necessarily be the most economic to manufacture. One way in which manufacturing constraints may be represented is to invoke them as boundary conditions. For example, the definition of a constant wall thickness, say associated with the manufacture of a component from sheet metal. The flexibility of SCOPES process easily facilitates such modelling.

However, sometimes it is may be very helpful to apply the process to an initial structure with no constraints. To let a user see what an idealised solution would look like in order to inspire new design ideas and the possibilities for innovative design solutions.

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