Applying CFD in the analysis of heavy oil - water two-phase flow in joints by using core annular flow technique

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ABSTRACT
In the oil industry the multiphase flow occur throughout the production chain, from reservoir rock until separation units through the production column, risers and pipelines. During the whole process the fluid flows through the horizontal pipes, curves, connections and T joints. Today, technological and economic challenges facing the oil industry is related to heavy oil transportation due to its unfavourable characteristics such as high viscosity and high density that provokes high pressure drop along the flow. The core-flow technique consists in the injection of small amounts of water into the pipe to form a ring of water between the oil and the wall of the pipe which provides the reduction of friction pressure drop along the flow. This paper aim to model and simulate the transient two-phase flow (water-heavy oil) in a horizontal pipe and T joint by numerical simulation using the software ANSYS CFX® Release 12.0. Results of pressure and volumetric fraction distribution inside the horizontal pipe and T joint are presented and analysed.

Keywords: Core-flow, pressure drop, numerical simulation, T joint

1. INTRODUCTION
According to late 2004 data, Brazil has a proven liquid oil reserve of 11 billion barrels, being 2.9 billion the amount relative to heavy oil, mostly located in offshore oil fields. This portion could double, in a medium period, with the implementation of new production projects that, if successful, could incorporate large volume reserves already discovered.

To generate projects, which ensure a significant volumetric recovery from reservoirs and improve existing projects, is of fundamental importance the development of new production technologies focused on heavy oil, especially in the scenario of offshore oil fields. The Brazilian oil (10 < API < 22) (American Petroleum Institute) have been classified by the ANP (National Agency of Petroleum, Natural Gas and Biofuels, Brazil) as heavy oil, whose density is similar to that of water (above 920 kg/m³). This is caused by the large proportion of hydrocarbons of high molar mass, typically with more than 15 carbon atoms per molecule, and large amounts of carbon residue, asphaltenes, sulfur, nitrogen, heavy metals, aromatics and/or paraffins [1]. Heavy oils present a high viscosity

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that ranges from 100 to 10000 times the viscosity of water, which makes it difficult, expensive and often unfeasible transporting the heavy oil from the reservoir to its destination through pipelines. This fact may lead to a lower productivity from the heavy oil reservoir, if compared with the production of light oils of Brent type. Moreover, production of this oil type requires a number of technological challenges.

Among the different methods for transporting heavy oils, we highlight the annular flow or Core Annular Flow (CAF), also known as core-flow. This technique basically consists of injecting small amounts of water causing the heavy oil to be enveloped by a layer of water and then heavy oil flows in the central region of the pipe without touching the inner wall of the pipe forming an oil core, thus providing an annular pattern. One disadvantage when using this technique occurs when the oil comes in contact with the inner wall of the pipeline during transport, as this can cause a big increase in system pressure and may cause serious damage to the transportation system, as well as serious environmental damage [2].

The interest in the production of heavy oil using the Core Annular Flow (CAF) technique has increased in recent years as a result of the large quantity of heavy oil reserves accessible. It is worth to emphasize that this technique has brought attractive results regarding energy consumption. This fact is due to reduction of pressure drop during the flow of water-oil type core-annular when compared with that which has to carry only the oil [3]. The Core Annular Flow technique does not modify the oil viscosity, but transforms the flow pattern and reduces friction during the flow [2, 4–9].

A theory for the stabilization of the annular pattern when two fluids of different viscosities and densities flow in a horizontal pipe was proposed by [2]. The theory is based on the analysis of the linear momentum equation in a transverse section of the pipe considering the effect of interfacial tension. This theory enabled the accomplishment of an interesting analogy between the peripheral flow and the flow bypassing a rising bubble, subsequently, observed by [10–12]. Bannwart [2] implies that the viscous and inertial forces on the annular flow can be combined into a single drag force alike the one observed in the bubbly flow. Ooms et al. [13] examined the annular flow in a stationary regime on a horizontal pipe and suggested a theoretical framework based on the theory of hydrodynamic lubrication. According to this model, we observed a harmonic motion in annular flow, i.e., the more viscous fluid (oil) moved on a corrugated form in the center of the horizontal pipe; such behaviour is known as Wavy Core-Annular Flow (WCAF).

Despite the studies of pressure drop, volume fractions between water and oil and shape of the water/heavy oil interface, few studies have been conducted to determine the influence of the wettability and the behaviour of different materials in the inner lining of pipes for transporting heavy oils. Studies concerning this subject were done by [14–26].

The Perfect Core Annular Flow (PCAF) it is a variation of the CAF. It occurs when the eccentricity is not verified in the flow. This flow pattern is seems to be very rare and can only exist for fluids of equal densities. Several experimental observations have shown that waves are formed in the interface water/oil leading to WCAF. The flow regime WCAF is observed in real situation [19].

Bai et al. [19] presented experimental results in vertical pipes lubricated by water with ascending and descending flow. According to these authors in ascending flow the oil tends to stay afloat concentrically to the axis of the pipe because of the center of gravity, they also identified a new flow pattern known as bamboo waves in the ascending flow and screw waves on descending flow. Based on the observations, they found that, for the given oil flow rate, there was a certain water flow in particular where the pressure drop was minimum. They also reported that the pressure drop, when the oil flowed alone, was about 200 times higher applying CFD in the analysis of heavy oil - water two-phase flow in joints by using core annular flow technique

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than in the case of flow using water as a lubricant for the same operating conditions and the same oil and pipe. In ascending flow, the pressure gradient and the buoyancy force have the same direction, waves develop and lubrication and buoyancy tend to extend the waves.

There are two factors that deserve care, regarding the performance of annular flow, and are related with the radial position of the oil core: the first is in relation to totally eccentric radial position and the second corresponds to concentric radial position. Rovinsky et al. [20] presented an analytical solution for two phase laminar flow with an eccentric Annular-Core configuration. According to the authors, the determination of the completely eccentric flow characteristics is important as a threshold for evaluating the effect of oil core eccentricity and as additional information as to the solutions of stratified flow with curved interface.

In a horizontal annular flow with density difference between the fluids, the oil core tends to occupy an eccentric position to the pipe axis and the presence of waves at the interface between oil and water induces a secondary motion perpendicular to the pipe axis.

Ooms and Poesio [13] report that this secondary movement is not considered in a concentric annular flow. Bentwich et al. [22] studied the eccentric annular flow, for the oil core and water pellicle flowing in laminar regime. The volumetric flow of two immiscible fluids was obtained by integrating the velocity profiles, in a horizontal pipe with a circular and eccentric interface. Curves were plotted of the pressure gradient reduction factor and the power reduction factor as a function of three parameters: ratio between the viscosity of heavy oil and water, the ratio between the diameters of the heavy oil core and the pipe and the viscosity. These authors concluded that for all oil viscosities and a given ratio between the diameters of the oil core and the pipe, the oil flow decreased with the increase of eccentricity of the heavy oil core. Moreover, for the given eccentricity, the oil flow reached a maximum value at a certain diameter ratio, which represented the optimum position of the oil-water interface.

Bensakhria et al. [4] evaluated the radial position of annular flow and showed that this position depends solely on the ratio of the contact perimeter (S) between the pipe wall and the fluid which forms the core (oil) and the perimeter of the pipe (S_o), i.e., \( \xi = S/S_o \). This ratio depends on the density difference between the fluids to be transported and lubrication, as well as the amount of water injected.

Ko et al. [9] used the finite element method for simulation of turbulent waves in the Core Annular Flow pattern, through the turbulence model \( \kappa-\omega \). They studied the behaviour of waves in terms of length, pressure gradient, the pressure distribution in the oil-water interface and the format of the waves varying with Reynolds number and the volumetric ratio between oil and water. According to authors, the wave forms computed and friction losses are in satisfactory agreement with experiments. According to Preziosi et al. [22] the disturbances of Core Annular Flow are stable when the wave lengths are infinitely small for Reynolds number tending to zero, and when the ratio between the radius of the pipe and the radius of the interface does not exceed a critical value which depends on the relation of the viscosity.

To correctly represent the pressure drop data, it is necessary to model the effects of WCAF such as turbulence and floatability. The term buoyancy favours the oil flow, but the same is affected by a corrugated interface in the oil-water two phase flow [8].

One of the central questions regarding the Core Annular Flow in a horizontal pipe is: how the drag force on the oil core, as the result of any density difference between oil and water, will be counterbalanced? Given this, a theoretical model was developed by [23] which gives a possible answer to this question. In this model it was assumed that the oil viscosity is so high that any change in the form of water-oil interface with time, can be neglected. Hence, it was
assumed that the oil core is a solid and thus the liquid-liquid interface became a solid-liquid interface. According to this model, a wavy motion of the oil core induces pressure variations in the pellicle formed by water, which can exert a force on the core in a vertical direction. This force can be so great that counterbalances the drag force on the core formed by oil, allowing a stable annular flow. The theoretical results coincided with the experimental results for a pipe with two inches in diameter, the oil viscosity ranging from 2.3 to 3.3 Pa.s and water as a lubricating fluid. However, for a pipe of eight inches in diameter with oil viscosity from 1.2 to 2.2 Pa.s, the calculated values differed more than 30% from measured values.

Brauner [24] proposed an analytical model to predict the holdup and pressure drop in situ for a horizontal pipe and obtained the power saving factor as a function to the viscosity ratio. They observed that in the case of laminar flow of two fluids, the energy saving factor is independent of the properties of fluids, and for turbulent flow, this factor increases with the decrease of density difference between the phases.

Arney et al. [25] performed experimental studies using the core-flow technique. The authors measured the pressure drop and the holdup with different rates of inflow and was suggested an empirical correlation for the holdup in terms of a water volumetric fraction. They also showed a curve of the friction factor as a function by the Reynolds number to perform a theoretical study of perfect annular flow. The curve may well predict the friction factor for high values of Reynolds number, but for low values did not obtained good results compared to the experimental data.

Based in explanation present in this paper, the present work aims to study numerically the transportation of heavy oils in horizontal pipe and T joints, using the technique of parietal lubrication by water (Core Annular Flow Technique) with the support of a computational tool (CFD).

2. METHODOLOGY
2.1. PHYSICAL PROBLEM DESCRIPTION
The physical problem evaluated in this work consists in a two-phase flow (water and heavy oil) in a horizontal pipe and T Joints both with 6 meters hydraulic length and 0.15 meters diameter. Figures 1 to 2 illustrate the representation the geometrical horizontal pipe and T Joints as well as details of geometry and mesh, respectively, used to study two-phase flow of water and highly viscous oil.

The meshes were created in module ANSYS ICEM CFD® Release 12.0. This domain of study was created by defining points, curves, surfaces and solids describing its size and shape. From the analysis of these figures we can observe the main details of the mesh near the wall of the horizontal pipe and T joint, close to the regions of interface, where the velocity gradients are most relevant. The refinement of this region was made and compared to the central region of the mesh in the search to get results closer to reality where the fluids has a well-defined interface.

It is through the interface that exists heat exchange, dissolution, drag, etc., i.e., the interface is the region of space where the different phases exchange information, and also where occurs transfer of heat, mass, and momentum. Further, in the Figures 1 and 2, we can see that exists two inlet of fluids: one for oil (central core) and another for water (annular region).

2.2. MATHEMATICAL MODELING
The modeling procedure consists in the mathematical description of the physical problem to be analyzed. In the case of fluid flow, the mathematical model consists of conservation
equations (mass, energy and momentum), initial and boundary conditions, and constitutive equations establishing the relationship between the stress and velocity in flow, among others. However, for engineering, these models correspond to a set of data and abstract ideas that allow the engineer, or researcher, to propose an explanation for the phenomenon that is being studied.

To model the transient multiphase flow, the following equations can be used:

2.2.1. Mass conservation equation
This equation is given as follows:

\[
\frac{\partial}{\partial t}(f_\alpha \rho_\alpha) + \nabla \cdot (f_\alpha \rho_\alpha \vec{U}_\alpha) = S_{MSa} + \sum_{\beta=1}^{N_s} \Gamma_{a\beta}
\] (1)

When the term of mass source \(S_{MSa}\) and the term of mass diffusivity \(\Gamma_{a\beta}\) are neglected, we can write the Equation 1 as:

\[
\frac{\partial}{\partial t}(f_\alpha \rho_\alpha) + \nabla \cdot (f_\alpha \rho_\alpha \vec{U}_\alpha) = 0
\] (2)
2.2.2. Momentum equation

This equation is given as follows:

$$\frac{\partial}{\partial t} (f_\alpha \rho_\alpha \tilde{U}_\alpha) + \nabla \cdot \left[ f_\alpha (\rho_\alpha \tilde{U}_\alpha \otimes \tilde{U}_\alpha) \right] = -f_\alpha \nabla \rho_\alpha + \nabla \cdot \left[ f_\alpha \mu_\alpha \left( \nabla \tilde{U}_\alpha + \left( \nabla \tilde{U}_\alpha \right)^T \right) \right]$$

$$+ \sum_{\beta=1}^{N} (\Gamma^+_{\alpha\beta} \tilde{U}_{\beta} - \Gamma^+_{\beta\alpha} \tilde{U}_\alpha) + \tilde{S}_{Ma} + \tilde{M}_\alpha$$

$$\text{(3)}$$

In the equations (1), (2) and (3), the sub-index $\alpha$ is the phase indicator on the two-phase water-heavy oil flow; $f$, $\rho$, $\mu$ and $\tilde{U}$ are the volume fraction, density, dynamic viscosity and
velocity vector, respectively; \( p \) is pressure and \( \mathbf{S}_{Ma} \) represents the term of external forces which act on the system per unit volume. In the term regarding momentum transfer induced by interfacial mass transfer (third term on the right side of equality) the sub-indexes \( \alpha \) and \( \beta \) correspond the phases involved, water and heavy oil. \( F_{a \beta} \) corresponds to the mass flow rate per unit volume of phase \( \alpha \) to phase \( \beta \) and vice-versa; \( \mathbf{M}_\alpha \) describes the total force per unit volume (interfacial drag force, lift force, wall lubrication force, virtual mass and turbulent dispersion force) on the \( \alpha \) phase due to interaction with \( \beta \) phase.

In this work the interfacial mass transfer term was not taken into account because the interfacial mass transfer in the momentum equation is used for disperse solid phase representing an additional force due to collisions between particles [29]. Thus, Equation 3 reduces to:

\[
\frac{\partial}{\partial t}(f_\alpha \rho_\alpha \mathbf{U}_\alpha) + \nabla \cdot \left[ f_\alpha (\rho_\alpha \mathbf{U}_\alpha \otimes \mathbf{U}_\alpha) \right] = -f_\alpha \nabla p + \nabla \cdot \left[ f_\alpha \mu_\alpha \left( \nabla \mathbf{U}_\alpha + (\nabla \mathbf{U}_\alpha)^T \right) \right] + \mathbf{M}_\alpha \\
\tag{4}
\]

2.2.3. Turbulence model

The turbulence model used for the water phase was the \( k-\varepsilon \) model. In this model, it is assumed that Reynolds tensors are proportional to the average velocity gradients, with the constant of proportionality being characterized by turbulent viscosity (idealization known as Boussinesq hypothesis).

The characteristic of this type of model is that two transport equations modeled, separately, are solved for the turbulent length and time scale or for which either two combinations are linearly independent. The transport equations for the turbulent kinetic energy, \( k \), and turbulent dissipation rate, \( \varepsilon \), are respectively:

\[
\frac{\partial (\rho_\alpha f_\alpha k_\alpha)}{\partial t} + \nabla \cdot \left[f_\alpha \rho_\alpha \mathbf{U}_\alpha k_\alpha - \left( \mu + \frac{\mu_\alpha}{\sigma_k} \right) \nabla k_\alpha \right] = f_\alpha \left( G_\alpha - \rho_\alpha \varepsilon_\alpha \right) \\
\tag{6}
\]

\[
\frac{\partial (\rho_\alpha f_\alpha \varepsilon_\alpha)}{\partial t} + \nabla \cdot \left[f_\alpha \rho_\alpha \mathbf{U}_\alpha \varepsilon_\alpha - \left( \mu + \frac{\mu_\alpha}{\sigma_\varepsilon} \right) \nabla \varepsilon_\alpha \right] = f_\alpha \frac{\varepsilon_\alpha}{k_\alpha} \left( C_1 G_\alpha - C_2 \rho_\alpha \varepsilon_\alpha \right) \\
\tag{7}
\]

where \( G_\alpha \) is the production of turbulent kinetic energy within the phase \( \alpha \); \( C_1 \) and \( C_2 \) are empirical constants. Although on these equations, the dissipation rate of turbulent kinetic energy and turbulent kinetic energy for phase \( \alpha \), are defined by:

\[
\varepsilon_\alpha = \frac{c_\mu S^3_{Ma \alpha}}{l_\alpha} \\
\tag{8}
\]
where \( l_\alpha \) is the length of spatial scale, \( q_\alpha \) is the velocity range and \( c_\mu \) is an empirical constant, given by:

\[
c_\mu = 4c_\alpha^2
\]  

where \( c_\alpha \) is also an empirical constant. The turbulent viscosity \( \mu_{\tau\alpha} \) is given as follows:

\[
\mu_{\tau\alpha} = c_\mu \rho_\alpha \frac{k_\alpha^2}{\varepsilon_\alpha}
\]  

The constants used on previous equations are: \( C_1 = 1.44; C_2 = 1.92; C_\mu = 0.09; \sigma_k = 1.0 \) and \( \sigma_\varepsilon = 1.3 \).

2.2.4. Mixture model (constitutive equations)

For this work we adopted the Eulerian model of mixing, for the analysis of water-heavy oil two-phase flow in pipes with curved connections. This model can present more complex formulations, however, is suitable for the modeling of liquid-liquid two-phase flow, where is calculated all forces acting on the interface in order to represent the water-heavy oil standard annular flow. In this model was considered only the total drag force exerted by the phase \( \beta \) under the phase \( \alpha \) per unit volume, as follows:

\[
\tilde{M}_\alpha = C_D \rho_{\alpha\beta} A_{\alpha\beta} \left( \tilde{U}_\beta - \tilde{U}_\alpha \right) \left( \tilde{U}_\beta - \tilde{U}_\alpha \right)
\]  

where \( C_D = 1.44 \) is the drag coefficient. The density and viscosity of the mixture are calculated respectively by:

\[
\rho_{\alpha\beta} = f_\alpha \rho_\alpha + f_\beta \rho_\beta
\]  

\[
\mu_{\alpha\beta} = f_\alpha \mu_\alpha + f_\beta \mu_\beta
\]  

The mixture model treats both phases \( \alpha \) and \( \beta \) symmetrically. The surface area per unit volume is given by:

\[
A_{\alpha\beta} = \frac{f_\alpha f_\beta}{d_{\alpha\beta}}
\]  

where \( d_{\alpha\beta} = 1 \text{ mm} \) is an interfacial length scale which must be specified.

The dimensionless transfer coefficient between the phases (Reynolds number of the mixture) is given by:
2.3. BOUNDARY CONDITIONS

In the inlet section for oil was used a prescribed value for the velocity component \( U_o = 1.0 \) m/s and oil volumetric fraction \( f_o = 1.0 \). Further,

a) \( u = v = 0 \) and \( w = U_o \) in \( z = 0 \) to \( \forall (x,y) \);

b) Laminar flow.

In the annular section for the inlet of water was used a prescribed value for the velocity component \( U_w = 1.8 \) m/s and water volume fraction \( f_w = 1.0 \). Further,

a) \( u = v = 0 \) and \( w = U_w \) in \( z = 0 \) to \( \forall (x,y) \);

b) Turbulent flow.

In the wall of the pipe was used no-slip condition and a roughness 45 \( \mu \)m. Thus,

a) \( u = v = w = 0 \) at the pipe wall.

Table 1 reports the physical properties of the fluids used in all simulation.

3. RESULTS AND DISCUSSIONS

Figure 3 shows the pressure field in horizontal pipe. It’s possible to see that the pressure is higher in the inlet and decrease along the pipe. This occurs due to pressure drop by friction that happen between the pipe wall and the fluid. The value of pressure drop was \( \Delta P = 5202 \) Pa.

Figures 4 and 5 show the pressure field in T junction and detail of the pressure field in the curvature region. From analysis of these figures we can see that the region that’s have the biggest value of pressure is in the intersection where fluids touch the walls between the vertical extension and horizontal extension.

The fact that the pipe curvature was smoothed may have been the cause of increase in

\[
\text{Re}_{\alpha\beta} = \frac{\rho_{\alpha\beta} |\vec{U}_\beta - \vec{U}_\alpha| d_{\alpha\beta}}{\mu_{\alpha\beta}}
\]

(16)

Table 1  The physical properties of the water and oil used in this work.

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Water</th>
<th>Heavy oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>997</td>
<td>989</td>
</tr>
<tr>
<td>Dynamic viscosity (Pa.s)</td>
<td>(8.89 \times 10^{-4})</td>
<td>10.0</td>
</tr>
<tr>
<td>Surface tension (N/m)</td>
<td>0.072</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3  Pressure field along the horizontal pipe.
pressure since the area of direct impact of fluid increased. Thus, friction has increased providing increased pressure at this location by friction.

Figure 6 illustrates the pressure difference as a function of time for the horizontal pipe and T junction. It’s possible to verify that the difference of pressure for T junction along the time is higher as compared with the horizontal pipe.

In this figure, we observe that the pressure drop increases over time, i.e., with increasing volume fraction of heavy oil in the pipe and T joint. In about two seconds after the starting time, the pressure drop increases slowly following the pressure drop rapidly increases to $\Delta P = 6654$ Pa ($t = 5.5$ s) in the horizontal pipe and $\Delta P = 5220$ Pa in T joint. At this point heavy oil now has advanced substantially for both the horizontal pipe and T joint. After 5 seconds, the pressure drop in the horizontal pipe rapidly decreases, because the water-oil two-phase flow reaches the steady state pressure drop and no more undergoes variation. At the T joint is different. In the time interval from 5 to 6 seconds, unlike what happens in the
Figure 6  Pressure drop in the horizontal pipe and in the T joint.

Figure 7  Heavy oil volume fraction distribution in the horizontal pipe at five different times.
flow in the curved pipe, the pressure drop at the T joint continues to increase until 8 seconds. After this time, the pressure drop no longer suffers variations, maintaining a pressure drop of \( \Delta P = 7324 \) Pa.

Figure 7 shows the heavy oil volume fraction in horizontal pipe to different times during the two phase flow. It’s possible to see the core annular flow pattern, where a thin water film flows near the wall pipe, while the heavy oil, flows in the middle of the pipe. Figure 7 shows yet the waves formatted during the flow. This waves happen because the phenomenal that’s occurs due to contact between heavy oil and water in interface region. Another phenomenon that can be

![Figure 8](image-url)
observed is the stratification of heavy oil and water due to difference in density of phases.

Figure 8 shows the heavy oil volume fraction distribution in the T joint at different times during the flow. At approximately 2.18 s the oil phase approaches to bend connection. In this region, the flow pattern is annular. Starting from this instant occurs a change of flow pattern from annular to stratified flow. So, the heavy oil flowing in both horizontal and vertical directions touch the upper wall of the pipe thus, unconfiguring the core-flow pattern and increasing the pressure drop. Now, after 10 s, we can see that the flow is in a steady-state condition.

4. CONCLUSIONS
After the 3D simulation of transient two-phase flow (heavy oil and water) in horizontal pipe and T junction the following conclusions can be made:
1. The ANSYS CFX was satisfactory to analyze the two phase flow (heavy oil and water) along the horizontal pipe and T joints.
2. In both two phase flow, horizontal pipe and T joint, it was observed a thin film of water near the wall and the core of heavy oil in the center of the pipe, i.e., the annular flow pattern was established.
3. The core-flow technique is efficient to flow the heavy oil in horizontal pipe because the flow pattern occurred along the time. The value of pressure drop in this case was $\Delta P = 5220\text{Pa}$.
4. The core-flow technique is not efficient to transport the heavy oil in T joint due to heavy oil touch the pipe wall during the flow. Consequently the flow pattern change from annular to stratified. In this case, the value of pressure drop was $\Delta P = 7324\text{ Pa}$, higher than the in horizontal pipe.

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