Modeling the thermal behavior of fluid flow inside channels using an artificial locally linear neuro-fuzzy approach

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ABSTRACT
Enhanced surface heat exchangers are commonly used all worldwide. If applicable, due to their complicated geometry, simulating corrugated plate heat exchangers is a time-consuming process. In the present study, first we simulate the heat transfer in a sharp V-shape corrugation cell with constant temperature walls; then, we use a Locally Linear Neuro-Fuzzy method based on a radial basis function (RBFs) to model the temperature field in the whole channel. New approach is developed to deal with fast computational and low memory resources that can be used with the largest available data sets. The purpose of the research is to reveal the advantages of proposed Neuro-Fuzzy model as a powerful modeling system designed for predicting and to make a fair comparison between it and the successful FLUENT simulated approaches in its best structures.

Keywords: Corrugated channel; Heat transfer; Prediction; Locally Linear Neuro-Fuzzy

1. INTRODUCTION
Plate heat exchangers (PHE) are replacing the tubular heat exchangers due to different advantages they offer such as operational flexibility, excellent heat transfer efficiency per unit volume and convenience in inspection, cleaning or repairing by adding or removing the individual plates.

Several flow patterns are possible for a PHE, depending on the exchanger configuration, which comprises a number of channels, pass arrangement, type of channel flow and the location of the inlet and outlet connections to the frame. Because of the large number of possible configurations and the vast variety of commercial plates, the design of the PHE is highly specialized.

Corrugated channels are a well-known category of PHEs, developed to improve heat transfer performance. Since these channels can lengthen the flow path and cause better
mixing, higher heat transfer performance is obtained compared to straight ducts. Corrugated duct is a good alternative for high heat flux applications or for more efficient heat exchange devices used in a wide variety of engineering applications like heating and air conditioning units. [1]

The most important parameters affecting heat transfer are aspect ratio and Reynolds number.

Artificial learning methodologies such as biologically motivated learning algorithms seem to be adequately powerful to be used in the proposed problem. Various numerical techniques have already been demonstrated for modeling of nonlinearity. In recent years, the most successful approaches used for modeling and prediction of nonlinear systems have been neural networks, Neuro-fuzzy models and biologically motivated learning algorithms such as genetic programming, evolutionary algorithms and reinforcement learning. Among these, neural network models have yielded better accuracies due to their nonlinear mapping capabilities.

In classic fluid dynamics, using the three main equations of mass, momentum and energy conservation, one might determine the velocity and temperature field via numerical methods. Numerical simulations and algorithms have shown good performance in most cases but they need powerful processors and large memories. The application of this solution procedure, if not impossible, might be very complicated and time-consuming.

The problems mentioned above and the succes of various models based on Neuro-Fuzzy approaches in predicting a vast number of parameters in many different areas motivated us to apply Neuro-fuzzy models to construct an advanced architecture for an artificial system of predicting the temperature field in a complicated geometry. Furthermore, the remarkable properties of Neuro-Fuzzy approaches, describing the highly nonliner and complicated interconections in the dynamical systems and their accuracy in the multi-objective problems have made them powerful methodologies in modeling complex dynamics.

In the present research, considering the two dimensional mass, Navier-Stokes and energy equations, we study the quasi periodic and unpredictable behavior of temperature field of fluid flow in corrugated channels. Firstly, we obtain the velocity and temperature fields of one cell of a corrugated channel using the FLUENT software. The numerical data would be arranged as a set of training data for the neural network. In this study we use a Locally Linear Neuro-Fuzzy method based on radial basis function (RBFs) to extract the simple and accurate dynamic model from the achieved numerical fields. In this way, we utilize an incremental learning algorithm named Model Tree Algorithm to learn our proposed Neuro-Fuzzy model. We show that by extracting the nonlinear dynamics of the equation using locally linear approach in neural network with fuzzy interface, a smart neural model is obtained. Section 2 includes a brief history of related works. Section 3 describes the governing equations for a V-shape corrugated channel defining three different boundary conditions. The new method and its learning algorithm are explained in section 4 while section 5 will cover up the results and comparisons based on the proposed method. Finally, remarkable points of the methodology are concluded in section 6.

2. RELATED WORKS
A large number of experimental and numerical studies have been conducted to investigate the pressure drop and heat transfer in corrugated channels. Some of these research works discuss the v-shape corrugated channels [2,3,4,5,6].

As a new aspect, Artificial Intelligence emerged more than fifty years ago and promptly found its application in various fields. By now, intelligent approaches have addressed both
fully-rational features [7] and sub-rational reasoning [8, 9]. Following the emergence of Artificial Neural Network (ANN) models, it was utilized in popularity as a prediction tool during the late 1980s. Their successors, Neurofuzzy models, were introduced very soon and have also been used. While the black box models, in nonlinear system identification, are argued to have drawbacks like unreliable extrapolation, unscalability and little understanding, they have shown remarkably better results in the modeling of complex processes, where the white box models, based on physical knowledge, can not precisely describe the highly nonlinear and complicated interconnections in the dynamical system. The accuracy of white box models is restricted by knowledge, while the accuracy of black box models is restricted by data.

Among all proposed methodologies, neural networks have shown good performance in capturing the nonlinearities in data [7], and are successfully used to model and predict different chaotic data sets. In this way, there are some individual studies especially on prediction of space weather indices [10] based on pipelined recurrent neural network [11,12,13] based on locally linear neuron-fuzzy model [14] based on new learning approach to Takagi Sugeno neurofuzzy models [15] based on a new spectral approach to analysis of data. The success of the intellectual methods based on brain learning in purposeful modeling of nonlinearities of different patterns motivated us to use the same approach to construct an advanced architecture for modeling nonlinear behavior of temperature of a fluid flowing in a channel with a particular geometry.

3. HEAT TRANSFER IN CORRUGATED CHANNEL
3.1. GOVERNING EQUATIONS
The governing conservation equations can be expressed in the following general form [6]:

\[ \frac{\partial (\rho u \phi)}{\partial x} + \frac{\partial (\rho v \phi)}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \phi}{\partial y} \right) + b \phi \]  

(1)

Where \( \phi \) stands for different dependent variables (u, v, T, k and \( \varepsilon \)) which have been explained in table1.

Reynolds number is defined as:

\[ Re = \frac{u D_h}{v} \]  

(2)

Where \( D_h \) is hydraulic diameter and is expressed as:

\[ D_h = \frac{4A}{P_w} \]  

(3)

The full development of heat transfer coefficients are evaluated from the measured temperatures and heat inputs.

Fully developed Nusselt number is defined as:

\[ Nu = \frac{hD_h}{k} \]  

(4)
where $k$ is the thermal conductivity.

There are different definitions for aspect ratio. The following correlation which is the ratio of width to length of one cell is chosen:

$$\gamma = \frac{H}{L} \quad (5)$$
There are many different types of designs either in shape (sinusoidal, v-shaped, irregular,...) or size in corrugated channels. Aspect ratio is a geometrical factor defined to compare two different corrugated channels. In the present study aspect ratio is considered to be 0.33.

3.2. GEOMETRY AND BOUNDARY CONDITIONS

The corrugated plate heat exchangers appear in different geometries regarding the shape of the upper and lower plates. In the present study, heat transfer in parallel sharp V corrugated plate heat exchangers would be discussed. The constant wall temperature boundary condition which takes place in phase changing heat exchangers has been chosen as a base for future studies. As discussed in many references, the flow becomes fully developed after 3 to 5 corrugations and we can consider the heat transfer in one and safely generalize it to the whole plate.

The following assumptions have been made:

- One cell of the corrugation would be studied.
- The fluid flowing inside the channel is air.
- Two dimensional conservation equations would be solved.
- The flow is assumed to be fully developed.
- The walls are in constant temperatures.
- The aspect ratio is \( \gamma = 0.33 \).
- The governing equations are valid all along the channel; this is an important factor in training concept.

The data has been derived from FLUENT software based on three different assumptions for boundary conditions. Table 2 shows the three groups of boundary condition for three sets of data.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>B.C. 1</th>
<th>B.C. 2</th>
<th>B.C. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Wall</td>
<td>350 K</td>
<td>400 K</td>
<td>320 K</td>
</tr>
<tr>
<td>Lower Wall</td>
<td>320 K</td>
<td>310 K</td>
<td>320 K</td>
</tr>
</tbody>
</table>

4. LOCALLY LINEAR NEUROFUZZY (LLNF) MODEL

This section introduces a Neuro-Fuzzy approach called LLNF (Locally Linear Neuro Fuzzy) based on an incremental learning algorithms initiated at 1993 [16,17].

Fuzzy systems have drawn a great attention because of their capability of translating expert knowledge expressed by linguistic rules into a mathematical framework. However, as the system complexity increases, reliable fuzzy rules and membership functions (MFs) become difficult to determine. A solution can be obtained by the combined use of fuzzy logic and Artificial Neural Networks (ANN). Because of their nature, these two methodologies can be easily integrated. Neuro-Fuzzy hybrid systems combine the advantages of fuzzy systems and ANN. The ANN learning provides a good tool to adjust the knowledge and to automatically generate additional fuzzy rules and membership functions. On the other hand, fuzzy logic enhances the generalization capability of ANN by providing more reliable outputs when extrapolation is needed beyond the limits of the training data.

Learning algorithm procedure as the challenging aspect of intelligent models, can affect the accuracy of results. As a matter of fact, all well designed models require the appropriate learning algorithm in order to be trained. In other words, after determining the structure of model, learning algorithms are being used to adjust parameters. In this study, a more recent use of fuzzy logic in intelligent neural network modeling is considered. A fuzzy...
implementation of locally linear model has resulted in the introduction of neurofuzzy locally linear networks, which have been used in many control applications successfully. Locally Linear Model Tree (LOLIMOT) is a learning algorithm for locally linear neurofuzzy model which has shown good performance and is characterized by high generalization property. Mathematical description of developing a Locally Linear Neuro Fuzzy Model and also the comprehensive stages of Locally Linear Model Tree learning algorithm are followed.

4.1. NEUROFUZZY MODEL
The fundamental approach with locally linear neurofuzzy (LLNF) model is dividing the input space into small linear subspaces with fuzzy validity functions. Any produced linear part with its validity function can be described as a fuzzy neuron. Thus the total model is a neurofuzzy network with one hidden layer, and a linear neuron in the output layer which simply calculates the weighted sum of the outputs of locally linear models (LLMs). The network structure of described model is depicted in Fig.2.

\[
\hat{y}_i = \omega_{i_0} + \omega_{i_1} u_1 + \omega_{i_2} u_2 + \cdots + \omega_{i_p} u_p, \quad \hat{y} = \sum_{i=1}^{M} \hat{y}_i \phi_i(u)
\]

where \( u = [u_1, u_2, \ldots, u_p]^T \) is the model input, \( M \) is the number of LLM neurons, and \( \omega_{ij} \) denotes the LLM parameters of the \( i \)th neuron. The validity functions are chosen as normalized Gaussians; normalization is necessary for a proper interpretation of validity functions.

\[
\phi_i(u) = \frac{j_i(u)}{\sum_{j=1}^{M} j_j(u)}
\]
Each Gaussian validity function has two sets of parameters, centers \( (C_{ij}) \) and standard deviations \( (\sigma_{ij}) \) which are the \( Mp \) parameters of the nonlinear hidden layer. Optimization or learning methods are used to adjust both the parameters of local linear models \( (\omega_{ij}) \) and the parameters of validity functions \( (C_{ij} \text{ and } \sigma_{ij}) \). Global optimization of linear parameters is simply obtained by Least Squares Technique. The complete parameter vector contains \( M(p+1) \) elements:

\[
\omega = \begin{bmatrix} \omega_{10} & \omega_{11} & \ldots & \omega_{1p} & \omega_{20} & \omega_{21} & \ldots & \omega_{M0} & \ldots & \omega_{Mp} \end{bmatrix}
\]

and the associated regression matrix \( X \) for \( N \) measured data samples is

\[
X = \begin{bmatrix} X_1 & X_2 & \ldots & X_M \end{bmatrix}
\]

\[
X_i = \begin{bmatrix} \phi_i(u(1)) & u_i(1)\phi_i(u(1)) & \ldots & u_p(1)\phi_i(u(1)) \\ \phi_i(u(2)) & u_i(2)\phi_i(u(2)) & \ldots & u_p(2)\phi_i(u(2)) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_i(u(N)) & u_i(N)\phi_i(u(N)) & \ldots & u_p(N)\phi_i(u(N)) \end{bmatrix}
\]

Thus

\[
\hat{y} = X\hat{\omega} \quad ; \quad \hat{\omega} = \left(X^TX\right)^{-1}X^Ty
\]

The remarkable properties of locally linear neurofuzzy model, its transparency and intuitive construction lead to the use of least squares technique for rule antecedent parameters and incremental learning procedures for rule consequent parameters, the advantages of which are considered in the third section.

We used two indices, Root Mean Square Error and Correlation Coefficient, in order to analyze and compare our results based on neuro-fuzzy model with FLUENT simulated outcomes.

The Root Mean Squared Error RMSE of prediction is evaluated by the equation:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y - \hat{y})^2}
\]
where $\hat{y}$ is the data value predicted by prediction program (out of $n$ sample cases); and $y$ is the FLUENT data value (out of $n$ sample cases).

So, the RMSE index ranges from 0 to infinity, with 0 corresponding to the ideal. This statistic is also known as the fit standard error and the standard error of the regression. It is an estimate of the standard deviation of the random component in the data, and is defined as

$$RMSE = \sqrt{MSE}$$ (14)

where MSE is the mean square error or the residual mean square. Furthermore, an MSE value closer to 0 indicates a fit that is more useful for prediction.

What is the Correlation Coefficient? As a concept from statistics, it is a measure of how well trends in the predicted values follow trends actual values or other simulated ones. It is a measure of how well the predicted values from a forecast model “fit” with the real-life data or data obtained in other calculations.

The quantity $r$, called the linear correlation coefficient, measures the strength and the direction of a linear relationship between two variables.

The mathematical formula for computing $r$ is:

$$r = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(\hat{y}_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2 \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}}$$ (15)

The correlation coefficient is a number between 0 and 1. If there is no relationship between the predicted values and the actual values, the correlation coefficient is 0 or very low (the predicted values are no better than random numbers). As the strength of the relationship between the predicted values and actual values increases so does the correlation coefficient. A perfect fit gives a coefficient of 1.0. Thus, the higher the correlation coefficient, the better it would be.

4.2. LEARNING ALGORITHM

The most important property of locally linear neurofuzzy model is that one can use some intuitive algorithms in training. The model starts as an optimal least squares estimation, and the new local linear models are created to reduce the prediction error. Locally Linear Model Tree (LOLIMOT) is an incremental tree-construction algorithm that partitions the input space by axis orthogonal splits. It implements a heuristic search for the rule premise parameters and avoids a time consuming nonlinear optimization ([16],[17],[18]).

The LOLIMOT algorithm is described in five steps according to ([17]):

1. **Start with an initial model**: start with a single LLM, which is a global linear model over the whole input space with $\Phi_1(u) = 1$ and set $M = 1$. If there is a priori input space partitioning, it can be used as the initial structure.
2. **Find the worst LLM**: Calculate a local loss function e.g. MSE for each of the $i = 1, \ldots, M$ LLMs, and find the worst performing LLM.
3. **Check all divisions**: The worst LLM is considered for further refinement. The hyper rectangle of this LLM is split into two halves with an axis orthogonal split. Divisions in all dimensions are tried, and for each of the $p$ divisions the following steps are carried out:
a. Construction of the multi-dimensional membership functions for both generated hyper rectangles;

b. Construction of all validity functions.

c. Local estimation of the rule consequent parameters for both newly generated LLMs.

d. Calculations of the loss function for the current overall model.

4. **Find the best division:** The best of the $p$ alternatives checked in step 3 is selected, and the related validity functions and LLMs are constructed. The number of LLM neurons is incremented $M = M + 1$.

5. **Test the termination condition:** If the termination condition is met, then stop, else go to step 2.

This algorithm trains the model automatically, and requires just one user defined parameter: the termination condition. To avoid over fitting and to provide maximum generalization, which is the most important property of a predictor, the termination condition is defined by checking the error index on validation sets. The optimal number of neurons is achieved when the error index on validation sets starts to increase.

### 5. RESULTS AND DISCUSSION

LLNF with LOLIMOT algorithm are applied to make a one step-ahead prediction of the temperature index based on data obtained from Fluent. The models are trained by a set of 10080 samples from the first half of the corrugation cell. Each set of data consists of temperature, vertical and horizontal velocities of 10080 points of the channel. The input regressor vectors are vertical and horizontal velocities along with the temperature of the previous node. In the first half, the LLNF learns the nonlinear interconnection between temperature and velocities i.e. it adjusts the weights; then in the second half, it uses the trained model to predict temperature for a known velocity field. The data obtained in the second half of the cell is called test set. This process has been repeated for three boundary conditions. The algorithms are executed on a personal computer with restrictions of computational power and memory capacity. MATLAB Software was chosen to implement the main program because of its flexibility and consistency to neuro fuzzy modules. When the network learns the nonlinear pattern of heat transfer, then finally, test data of temperature field would be compared with the same values simulated by FLUENT.

In order to evaluate the generalization of the method it was applied to three sets of data with different boundary conditions.

The Root Mean Square Errors and also correlation coefficients of predicting Temperature by LOLIMOT in accordance with each three boundary conditions are shown in Table 3. It’s observed that all the prediction errors are tolerable and the performance of LOLIMOT in

<table>
<thead>
<tr>
<th>Three Boundary Conditions</th>
<th>RMSE Train</th>
<th>RMSE Test</th>
<th>Correlation Test</th>
<th>Correlation Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.C. 1</td>
<td>0.43</td>
<td>3.15</td>
<td>0.98</td>
<td>0.988</td>
</tr>
<tr>
<td>B.C. 2</td>
<td>0.588</td>
<td>0.88</td>
<td>0.981</td>
<td>0.969</td>
</tr>
<tr>
<td>B.C. 3</td>
<td>0.173</td>
<td>0.183</td>
<td>0.994</td>
<td>0.991</td>
</tr>
</tbody>
</table>
predicting temperature field is satisfactory. Besides, high correlation coefficients between predicted values and FLUENT values indicated that both data are statistically correlated.

Fig.3 to Fig. 5 show the values simulated by FLUENT and values predicted by LOLIMOT based predictor according to three boundary condition groups of temperature. Every one of three figures show temperature changes in point of \(x=0.033\) for train set and point of \(x=0.059\) for test set, associated with different height of channel. Fig.6 is the curve of predicted values vs. values simulated by FLUENT for test data of all three groups of boundary condition.

6. LIMITATIONS
Considering general restrictions in artificial models, most researches have focused on fully rational methods; however, nothing can alter the fact that uncertainties in real world data always affect the optimality of results. Although intellectual systems have been developed and used successfully, a new attitude towards sub-rational approaches has recently been proposed in order to overcome ambiguities.
Narrowing down to Neuro-Fuzzy models, an important point is the limitation of resources. The model trained by limited observations would have a weak generalization property. On the other hand, any attempt to predict real world phenomena usually necessitates multi objective learning procedures. Most important subsequence of multi objective learning, especially in purposeful models is high complexity and high computational requirement. This will significantly affect the real-time problems.

A further challenging point regarding proposed models is the length of the step in prediction. Despite the fact that intelligent black box models comprise neural and their successor neuro-fuzzy models have shown great accuracy in short term prediction, they reveal noteworthy inaccuracies in long term performance. The performance of long term prediction based on proposed model could be improved using decomposition analysis such as singular spectrum analysis or independent component analysis.

Figure 4 Prediction of Temperature With second boundary condition data; Upper: FLUENT and predicted values of Train data, Lower: FLUENT and predicted values of Test data.
7. CONCLUSION

In this study, a new methodology motivated by the concept of artificial intelligence, has been used to determine air temperature field inside a v-shape corrugated channel. We have used a Locally Linear Neuro-Fuzzy approach to achieve that goal. The temperature values simulated by FLUENT have been used as training data. Finally, test data have been compared with the same values. This has been achieved using horizontal and vertical velocities besides previous trends of air temperature inside the corrugated channel assuming three different boundary conditions. The main achievements reported in this paper can be summarized as follows:

![Graph showing predicted and Fluent data for temperature and height of channel under different boundary conditions.]

Figure 5  Prediction of Temperature With third boundary condition data; Upper: FLUENT and predicted values of Train data, Lower: FLUENT and predicted values of Test data.
The proposed model predicts the temperature field in a corrugated channel via a faster method with agreeable error. Temperature field in the second half of a corrugation cell was predicted and compared to the available data. This comparison was shown as the standard error RMSE and correlation factor which shows the good agreement between the two groups. While needing a lot of data points for training network has been considered as a limitation in using artificial models, the main problem is low prediction accuracy alongside the wall. The largest errors are met in the data near the walls because of the most nonlinear part in the temperature field. Using the proposed model to predict heat transfer in a complete corrugated channel or forecasting heat transfer in corrugated channels with different aspect ratios is anticipated for future work. Also developing a similar model to predict heat transfer in all corrugated channel with constant wall temperature would be so applicable.

Figure 6 Predicted data values vs. values simulated by FLUENT according to three boundary condition data.
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**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>A</td>
<td>Area</td>
</tr>
<tr>
<td>dP</td>
<td>Pressure gradient</td>
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<tr>
<td>(D_h)</td>
<td>Hydraulic diameter</td>
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<tr>
<td>f</td>
<td>Friction factor</td>
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<tr>
<td>h</td>
<td>Heat transfer coefficient</td>
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<tr>
<td>H</td>
<td>Channel height</td>
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<tr>
<td>k</td>
<td>Conductivity</td>
</tr>
<tr>
<td>L</td>
<td>Channel length</td>
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<tr>
<td>Nu</td>
<td>Nusselt number</td>
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<tr>
<td>P</td>
<td>Pressure</td>
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<td>(P_w)</td>
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<td>Pr</td>
<td>Prandtle number</td>
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<tr>
<td>R</td>
<td>Radius of curvature</td>
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<td>Re</td>
<td>Reynolds number</td>
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<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>u</td>
<td>Horizontal velocity</td>
</tr>
<tr>
<td>v</td>
<td>Vertical velocity</td>
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<td>x</td>
<td>Horizontal dimension</td>
</tr>
<tr>
<td>y</td>
<td>Vertical dimension</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Aspect Ratio</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Viscosity</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Corrugation angle</td>
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<tr>
<td>(\rho)</td>
<td>Density</td>
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**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
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<tr>
<td>b</td>
<td>Bulk</td>
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<tr>
<td>FD</td>
<td>Fully developed</td>
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<tr>
<td>m</td>
<td>Mean</td>
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<tr>
<td>w</td>
<td>Wall</td>
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