An empirical approach to river bed degradation

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ABSTRACT
Degradation in a homogeneous alluvial channel due to complete stoppage of sediment downstream of a high capacity reservoir has been studied empirically. Based on dimensional analysis, a method has been established for predicting transient bed profiles in degrading alluvial stream. In addition, prediction procedures for other degradation elements like maximum degradation at any time at upstream section and degradation extent has also been evolved. The prediction procedures when tested against known data gave satisfactory results.

Keywords: River bed degradation, empirical model, alluvial channel

1. INTRODUCTION
A hydraulic engineer, entrusted with the responsibility of developing the water resources and of using the alluvial streams for the benefit of mankind, has to deal with several complex problems. One such problem arises from the construction of a high capacity reservoir on a heavily sediment laden alluvial channel. A major part of the sediment being transported settles down in the reservoir, causing the overflow to be essentially clear or at least sediment-free.

The sediment-free outflow picks up material from the bed if the banks are nonerodible, thereby, lowering the bed level. This lowering of the bed level is called degradation or retrogression. Usually, the degradation process is quite slow. However, if it goes on unchecked, in extreme cases, it may become harmful to the structure itself. A striking example of this was the failure of Islam barrage on the Sutlej. Hence, the prediction of river bed degradation below a hydraulic structure, such as grade control structure, is very important for evaluating the safety of structure as well as for assessing the environmental impact of degradation on the riparian system along the river bank and the aquatic species.

Cases of degradation below dams, where most or all of the sediment discharge is retained in the reservoir, have been reported frequently in technical publications (Lane 1934; Shulits 1934; Hathaway 1948; Joglekar and Wadekar 1951; Stanley 1951; Pick 1951; Malhotra 1951; Livesey 1963). After the first significant experimental work on degradation due to sediment diminution was conducted by Harrison (1950), investigators have studied the degradation problem experimentally [Newton, 1951; Garde and Hasan, 1967; Suryanarayana and Shen, 1971; Aksoy, 1970, 1971], analytically [Culling (1960); Ashida and Michiue, 1971; DeVries, 1973; Vittal and Mittal, 1980, Begin, Meyer and Schumm, 1981] and numerically [Tinney, 1962; Chang, 1969; Gessler, 1971; Mehmood, 1975; Puls et al. 1977;
Simons et al. 1980; Jarmillo and Jain, 1984; Palaniappen, 1991; Yen et al., 1992], to name a few. However, all these studies do not help to understand the mechanism of the degradation problem which depends on such a large number of variables that they make the problem very complex.

Drew (1979), while giving the dynamic model for channel bed degradation, made a general statement that due to complexity of the physical situation all studies must resort to empiricism at some stage in the development. And in that case the reliance on empiricism replaces the description of some mechanical aspects of the problem.

Keeping this in view, an empirical solution of the problem of degradation in an homogeneous alluvial channel due to complete stoppage of sediment downstream of a high capacity reservoir, i.e. 100% trap efficiency (Fig. 1) has been attempted in this paper. It is hoped that the techniques developed would be fairly useful for other types of boundary conditions also.

2. THEORETICAL CONSIDERATIONS
The solution of the degradation problem involves the determination of three important elements of the degradation phenomenon which need to be understood and determined quantitatively. These are (i) the degradation depth at any distance, \( x \), and time, \( t \); (ii) degradation extent or in other words the length of transient bed profile for any time; and (iii) maximum degradation at any time at \( x = 0 \).

However, this problem is complicated by such inter-related variables as channel geometry and irregularities, type of bed and bank material, rate of bed material discharge, characteristics of channel alignment and slope, temperature of water-sediment complex flowing in the channel, etc. Hence, it is difficult to attempt a complete representation of the problem of degradation arising from the complete stoppage of sediment supply as it exists in nature, because if any of the significant variables has been neglected, the analysis will not yield proper results and if any redundant variable is included, it would unnecessarily complicate the analysis.
In order to make some headway in understanding the mechanism of degradation, certain plausible simplifying assumptions have been made in the present study. (i) the sediment is arrested completely by the reservoir; (ii) variation in water discharge does not occur; (iii) the sediment moves only as bed load and the river bed material is homogeneous with depth, i.e. no armoring is expected to take place, (iv) the river reach is straight; the banks are not erodible, (v) the river section approximates a rectangle and the influence of changes in bed forms on the resistance is neglected, (vi) one-dimensional treatment has been considered and (vii) there is no vegetation growth and no tributary flow into the main channel.

2.1. DEGRADATION DEPTH, Z

From the above listed variables affecting degradation at any time and distance, it can be seen that:

\[ z = f(L, B, h, U_m, \rho, g, \mu, G_c, z_0, x, t). \]  

(1)

Since, for uniform flow conditions, \( G_c = aU_m^b \), one can take \( G_c \) to be the unique function of \( U_m \), so that Eq. (1) becomes:

\[ z = f(L, B, h, U_m, \rho, g, \mu, z_0, x, t) \]

(2)

Since, there are total 11 numbers of variables and taking 3 variables \( h, U_m \) and \( \rho \) as repeated variables one can write using the Buckingham pi theorem:

\[ \frac{z}{h} = \left( \frac{z_0}{h} \frac{x}{L} \frac{B}{h} \frac{U_m t}{h} \frac{U_m}{\sqrt{g h}} \frac{U_m h}{\mu} \right) \]  

(3)

For a river channel of constant width, Eq. 3 becomes

\[ \frac{z}{h} = f \left( \frac{z_0}{h} \frac{x}{L} \frac{U_m t}{h}, F_r, R_e \right) \]  

(4)

Since in case of water flow dealing with bed movement \( R_e \) number is not effective, hence, can be neglected. Also for the particular flow conditions of water as Froude number will also not change, hence its effect can be neglected in the analysis, so that Eq. 4 reduces to:

\[ \frac{z}{h} = f \left( \frac{z_0}{h} \frac{x}{L} \frac{U_m t}{h} \right) \]  

(5)

Which can further be modified to:

\[ \frac{z}{z_0} = f \left( \frac{x}{L}, \frac{U_m t}{h} \right) \]  

(6)

2.2. MAXIMUM DEGRADATION DEPTH, \( Z_0 \):

Keeping above points in view, various parameters influencing maximum degradation, \( z_0 \) at the toe of the dam i.e. at \( x = 0 \) can be selected as: depth of flow, \( h \); velocity of flow, \( U_m \);
water density, $\rho$; dynamic viscosity of fluid, $\mu$; acceleration due to gravity, $g$; mean sediment size, $d_{50}$ and time, $t$.

With the assumptions and conditions of the present study, one can write:

$$z_0 = f(h, U_m, t, d_{50})$$  (7)

Using the Buckingham pi theorem, following can be written:

$$\frac{z_0}{h} = f\left(\frac{U_m t}{h}, \frac{d_{50}}{h}\right)$$  (8)

which can be modified further to:

$$\frac{z_0}{d_{50}} = f\left(\frac{U_m t}{h}\right)$$  (9)

2.3. EXTENT OF DEGRADATION, L
Similar to maximum degradation at $x = 0$, the various parameters influencing the extent of degradation, $L$, can be taken as depth of flow, $h$; velocity of flow, $U_m$; water density, $\rho$; dynamic viscosity of fluid, $\mu$; acceleration due to gravity, $g$; mean sediment size, $d_{50}$ and time, $t$. Hence using the Buckingham pi-theorem and modifying further, one can write:

$$\frac{L}{d_{50}} = f\left(\frac{U_m t}{h}\right)$$  (10)

3. DATA USED
In literature [Newton (1951), Suryanarayana and Shen (1969)], sufficient experimental data are already available on the problem of degradation arising due to complete stoppage of sediment supply at the upstream boundary. Hence, it was not thought necessary to do so further experimentation. Newton (1951) conducted four experimental runs on a 30 ft long, 1.0 ft wide and 2.0 ft, deep recirculating open channel laboratory flume using uniform sediment size 0.69 mm. Bhamidipaty and Shen (1969) conducted seven experimental runs for degradation on a recirculating tilting flume of a rectangular section 60 ft long, 2 ft wide and 2.5 ft deep with side walls made of plexiglass and bottom of stainless steel plates using 0.45 mm diameter particle size. In these experimental investigations, uniform flow

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth, ($h$)</td>
<td>m</td>
<td>0.0336-0.1586</td>
</tr>
<tr>
<td>Mean Velocity, ($U_m$)</td>
<td>m/sec</td>
<td>0.375-0.586</td>
</tr>
<tr>
<td>Slope ($S_o$)</td>
<td></td>
<td>0.0014-0.0066</td>
</tr>
<tr>
<td>Temperature, $T$</td>
<td>°C</td>
<td>68-70</td>
</tr>
<tr>
<td>Water Discharge ($Q$)</td>
<td>m$^3$ sec/m</td>
<td>$1.135 \times 10^{-1}$ - $3.90 \times 10^{-2}$</td>
</tr>
<tr>
<td>Sediment Transport rate ($G$)</td>
<td>(m$^3$sec$^{-1}$m$^{-1}$)</td>
<td>$3.289 \times 10^{-6}$ - $1.997 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
conditions were first established for a prescribed discharge and slope. The bed surface and water surface were measured with the help of a gauge mounted on a carriage which could travel along the rails mounted on the edge of the flume. While Newton (1951) measured bed surface with disc tapped indicator, Surayanarayana and Shen (1969) measured bed elevations with a point gauge with flat bottom having least count of 0.001 ft. The bed and water surface elevations were noted at various time intervals, gradually increasing with time. The details of the experimental set-up and procedure are given elsewhere [Newton (1951), Surayanarayana and Shen (1969)].

Out of seven degradation runs made by Surayanarayana and Shen (1969), 6 runs, namely Runs No. 22, 24, 26, B-5, B-6 and B-7, were selected because other runs seems to be more or less local scour phenomena instead of degradation and hence were not considered in the present analysis. The data available were in FPS system and since the empirical consideration includes dimensionless equations, it was used as it was. However, where needed it was converted into metric system. The ranges of various variables as used in the present study taken from the above investigations are shown in Table 1.

4. ANALYSIS OF DATA
4.1. TRANSIENT BED AND WATER SURFACE PROFILES
Bed and water surface elevations of selected runs were plotted on graph sheets as shown in Fig. 2 which is a typical plot of transient bed and water surface profiles for Run No. 22 of Surayanarayana and Shen (1969). The fluctuation of data indicates the presence of undulations (ripples/dunes) on the bed, which necessitates the averaging of the flow as is done in Fig. 3 of Run No. 2 of Newton (1951)

It can be seen from these types of plots that the shape of the transient bed profile is concave downwards and the ordinates of the curve are monotonically decreasing with increase in the distance. However, degradation was taken to end at a section beyond which the difference in the value of \( z \) between two consecutive sections was so small that it could not be distinguished in the plots. This define the location of degradation front and hence the length of degradation at various times. In the analysis which follows only those profiles were considered which do not reach the downstream; this makes the analysis semi-infinite. The degradation and flow depths at various sections in the longitudinal direction were taken from such averaged profiles and were used in the analysis.

Since the side walls in the experiments of Newton (1951), Suryanarayana (1969) were made of plexiglass and hence considerably smooth than the bed, the hydraulic radius with respect to the bed, \( R_b \), was calculated in place of \( h\). \( R_b \) can be found out by relation given in Garde and Rangaraju (1990).

\[
R_b = \left(1 - \frac{2R_w}{B}\right)h
\]  
\[\text{(11)}\]

Where,
\[
R_w = \left(\frac{U_m\eta}{S_e^{3/2}}\right)^{3/2}
\]
\[\text{(12)}\]

Here, \( R_w = \) Hydraulic radius of the wall.

4.2. DETERMINATION OF DEGRADATION DEPTH, Z
Figure 2  Typical Degradation original data [Run no. 22, Suryanarayana and Shen (1969)].

Figure 3  Transient bed and water surface profiles [Run No. 3, Newton (1951)].
The dimensionless parameters $z/z_0$ and $x/L$ of Eq. 6 (with $h$ replaced by $R_b$) were computed for all degradation runs of Newton (1951) and Suryanarayana (1969) and were plotted (not shown here). It was found that the third parameter $U_m t/R_b$ systematized the scatter on these plots. Then the following ranges of parameters $U_m t/R_b$ were selected: 10,000–40,000; 40,000–70,000; 70,000–1,00000; 1,00000–2,00000 and 2,00,000–3,50,000. The data was grouped into the above five ranges. The plots of $Z/Z_0$ VS $X/L$ for the individual range of were made, as shown in Fig. 4. Meanlines were drawn through these points on the figure. These graphs show that with the increase of distance and time the depth of degradation decreases. These five curves were carried over to a master plot of degradation as shown in Fig. 5. The mean lines have been found to follow error function and the following equation was found to fit these mean lines.

$$\frac{z}{z_0} = \left[1 - \text{erf} \left( \frac{2x}{L} \right) \right]^A$$

(13)

In this equation, $A$ is function of $U_m t/R_b$. Hence the values of $A$ fitting the five curves were plotted against $U_m t/R_b$ as shown in Fig. 6. It is interesting to note that the exponent decreases...
with the increase in \( \frac{U_m t}{R_b} \) ranges. The data well described by equation as:

\[
A = 1.021 - 4.521 \times 10^{-6} \left( \frac{U_m t}{R_b} \right) + 1.756 \times 10^{-11} \left( \frac{U_m t}{R_b} \right)^2
- 2.619 \times 10^{-17} \left( \frac{U_m t}{R_b} \right)^3
\]

Knowing the extent of degradation length \( L \), the maximum degradation depth \( z_0 \) and the value of \( A \) for the particular time \( t \), mean velocity \( U_m \) and hydraulic radius \( R_b \), the degradation depth \( z \) can be obtained from Eq. 13.

### 4.3. Determination of Maximum Degradation Depth, \( z_0 \):

The dimensionless parameter \( \frac{z_0}{d_{s0}} \) and \( \frac{U_m t}{R_b} \) (with \( h \) replaced by \( R_b \)) of Eq. 9, were computed for all degradation runs of Newton (1951) and Suryanarayana (1969) and are plotted as shown in Fig. 7. It can be noted that the maximum depth of degradation \( z_0 \) increases with the increasing time. The following equation was found to fit the data well:

\[
\frac{z_0}{d_{s0}} = 4 \times 10^{-5} \left( \frac{U_m t}{R_b} \right)^{1.24}
\]
Figure 6  Variation of $A$ with $U_{mt}/R_b$.

Figure 7  Variation of $z/d_{50}$ with $U_{mt}/R_b$.  

$A = 1.021 - 4.521 \times 10^{-6} (U_{mt}/R_b) + 1.75 \times 10^{-11} (U_{mt}/R_b)^2 - 2.619 \times 10^{-17} (U_{mt}/R_b)^3$

$Z_0/d_{50} = 4 \times 10^{-5} (U_{mt}/R_b)^{1.24}$

$R = 1.30$
4.4. DETERMINATION OF EXTENT OF DEGRADATION, L

Equation 13 will be useful if the extent of degradation $L$ is known. The dimensionless parameter $\frac{L}{d_{50}}$ and $\frac{U_m t}{R_b}$ (with $h$ replaced by $R_b$) of Eq.10, were computed for all degradation runs of Newton (1951) and Suryanarayana (1969) and are shown in Fig. 8. The data are well described by the following equation:

$$\frac{L}{d_{50}} = 54.23 \times \left( \frac{U_m t}{R_b} \right)^{0.49}$$

(16)

4.5. STEPS FOR DETERMINATION OF TRANSIENT BED PROFILES BY EMPIRICAL MODEL

For the known values of time $t$, mean velocity $U_m$, sediment size $d_{50}$ and hydraulic radius $R_b$ the transient bed profile can be predicted as given below:

1. Determine extent of degradation $L$, from the Eq. 16.
2. Determine the value of maximum degradation depth $z_0$, from the Eq. 15.
3. Determine the value of $A$ for the known value of $\frac{U_m t}{R_b}$, from Eq. 14.
4. Compute degradation depth $z$, at the Evarious sections for the time $t$ with the computed values of $L$ and $z_0$ (as defined above) from the Eq. 13.

With the help of above mentioned steps, the transient bed profiles for the numerical data of Gessler (1971) were computed and plotted against the observed profile as shown in Fig. 9. The calculated data by the model fall within $\pm 40\%$ error line. Considering uncertainties in the
measurements and the limitations of Gessler (1971) numerical model, the estimates of bed degradation by the proposed empirical relationships are considered to be satisfactory.

5. CONCLUSION
An empirical method based on dimensional analysis has been established for predicting transient bed profiles in a degrading stream due to complete stoppage of sediment supply. This method yields quick and reasonable estimates of the extent and magnitude of degradation due to 100% sediment deficit once the measurable and quickly determinable parameters, such as velocity of flow and depth of flow, are known.

REFERENCES

Figure 9 Comparison of observed and calculated values of z for Gessler(1971) data.


**NOMENCLATURE**

- $A_s$: shape coefficient of transient bed profile
- $F$: Froude number
- $g$: acceleration due to gravity
- $\Delta G$: sediment deficit
- $G_e$: equilibrium sediment transport rate
- $h$: mean depth of flow
- $L$: degradation extent
- $Q$: water discharge
- $R_b$: hydraulic radius corresponding to bed
- $S$: slope
- $t$: time
- $T$: temperature
- $U$: velocity of flow
- $x$: distance
- $Z$: degradation depth at any time $t$ and at any distance $x$
- $Z_o$: maximum degradation depth at any time $t$ and at $x = 0$